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FALL TERM
SELF-STUDY

Preliminaries

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, the actual homework assignments are stated. Please, hand-in your results of the homework assignments through MSTeams (see there for further details).

Additional Exercises (see the lecture slides for solutions):

Exercise 1.1: Evaluate these expressions (without using your calculator):

$$\textbf{a)}\quad 8^{-1/3}\,, \qquad \textbf{b)}\quad \left(\tfrac{1}{100}\right)^{-3/2}\,, \qquad \text{and} \qquad \textbf{c)}\quad 5^0\,.$$

as well as

$$\textbf{d)}\quad (2^{-2})^3\,, \qquad \textbf{e)}\quad \frac{3^3}{3^{1/3}\cdot 3^{2/3}}\,, \qquad \text{and} \qquad \textbf{f)}\quad 2^{7/4}\cdot 8^{-1/4}\,.$$

Exercise 1.2: Simplify the expression:

$$\frac{4(x+3)^4(x-2)^2 - 6(x+3)^2(x-2)^3}{(x+3)(x-2)^3}$$

Exercise 1.3: Solve the quadratic equations (where possible):

$$\textbf{a)}\quad x^2+3x+1 \;=\; 0\,, \qquad \text{and} \qquad \textbf{b)}\quad x^2+x+1 \;=\; 0\,.$$

Exercise 1.4: Factor the polynomials

- **a)** $p(x) = 12x^2 - 11x - 15$ by using the solution formula for quadratic polynomials.
- **b)** $p(x) = 4(x-2)^3 + 3(x-2)^2$ by grouping terms strategically.
- **c)** $p(x) = 9x^2 - 49$ by grouping terms strategically.

Exercise 1.5: Give the following as a rational expression in lowest terms:

$$\left(\frac{x^3 - 7x^2 + 10x}{x^2 + 6x + 9}\right)\left(\frac{x+3}{x-5}\right)\,.$$

Homework Assignment:

Problem 1.1: Elementary Algebra - Part I.

a) Solve $\frac{3}{2} + \frac{5}{4}$.

b) Factor $x^2 + 3x$.

c) Solve $(-6)(-3) + 8(-1)$.

d) Solve $x^3 \cdot x^4$.

e) If a bag of sand weights 80 kilograms, then how much kilograms are 5% of the bag.

f) Solve $[(3x-1)/(2x)] - \frac{1}{2}$ (*Hint*: bring to a common denominator).

g) Solve for x: $6x + 2 = -x - 1$.

h) If $x = 3$ and $y = 9$, then $\sqrt{y}/3x$.

i) Simplify $(x^2 - 16)/(x - 4)$.

Problem 1.2: Elementary Algebra - Part II.

a) What is the slope of the line $3y + 4x = 2$?

b) For which values of x is $3x + 2 > 0$?

c) For which values of x is $3x^2 - 2x - 1 > 0$?

d) Solve for x: $2x^2 + 8x - 11 = 0$.

e) Sketch the graph of $f(x) = x^2 - x - 2$.

f) For what values of x is $3x + 2 > 2x - 8$?

g) Where does the graph of $f(x) = 3x - 2$ intersect the graph of $g(x) = x^2$?

h) Let

$$g(x) \;=\; \frac{3x^2 + 2x - 8}{2x^2 - x}\,.$$

Compute $g(2)$ and state the domain of g.

i) Sketch the curve $x^2 + y^2 - 2x - 4y + 1 = 0$.

j) Find the distance between the point $(1, 1)$ and the intersection point of the lines $y = -2x + 1$ and $y = 4x - 5$.

Problem 1.3: Elementary Algebra - Part III.

a) Solve the inequality in terms of intervals and illustrate the solution set on the real number line.

$$\textbf{i)}\quad \tfrac{1}{x} \;<\; \sqrt{16}\,, \qquad \textbf{ii)}\quad |5x - 2| \;<\; 6\,,$$

as well as

$$\textbf{iii)}\quad x^2 \;<\; 2x + 2^3\,, \qquad \text{and} \qquad \textbf{iv)}\quad 4x \;<\; 2x + 1 \;\leq\; 3x + 2\,.$$

b) Solve the following equations (where possible):

$$\textbf{i)}\quad x^2 - 7x - 18 \;=\; 0\,, \quad \textbf{ii)}\quad x^2 - 9x + 3 \;=\; -5\,, \quad \text{and} \quad \textbf{iii)}\quad 28x^4 + 16x^3 \;=\; 80x^2\,.$$

c) Solve the following equations applying the laws of exponential expressions and logarithms:

$$\textbf{i)}\quad 2^x \;=\; 74\,, \quad \textbf{ii)}\quad 6^{4x-17} \;=\; 216\,, \quad \text{and} \quad \textbf{iii)}\quad 2 \cdot \log(8x + 4) + 6 \;=\; 10\,.$$

d) Simplify the fractions by applying polynomial long division and give (when applicable) a remainder term in lowest terms.

$$\textbf{i)}\quad \frac{6x^2 - 25x - 25}{6x + 5}\,, \qquad \textbf{ii)}\quad \frac{5x^4 - 3x^2 + 2}{x^2 - 3x + 5}\,,$$

as well as

$$\textbf{iii)}\quad \frac{3x^2 + 23x + 14}{x + 7}\,, \qquad \text{and} \qquad \textbf{iv)}\quad \frac{2x^5 - 3x^4 - x^2 + x + 4}{x^2 + 1} \;\leq\; 3x + 2\,.$$

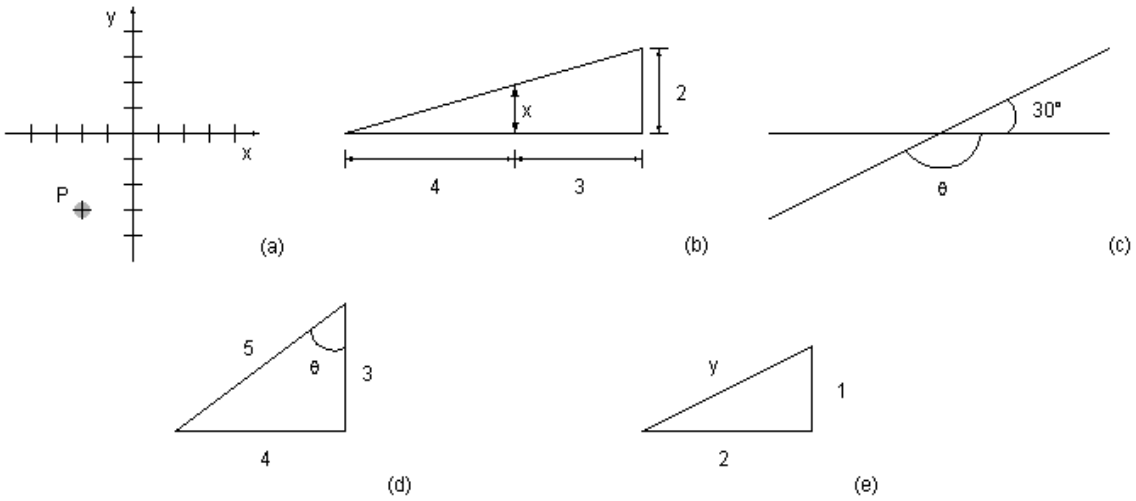

Figure 1: Sketches of the situations addressed in problem 1.4.

Problem 1.4: Recap: Elementary Geometry and Trigonometry.

a) Give the coordinates of the point P shown in Fig. 1 (a).

b) Find x as indicated in Fig. 1 (b).

c) Find θ as indicated in Fig. 1 (c).

d) Find $\sin(\theta)$ as indicated in Fig. 1 (d).

e) Find y as indicated in Fig. 1 (e).

f) What is the circumference of a circle whose radius is 4 centimeters?

g) A rectangle has an area of 10 square meters and one side of length 5 meters. What is its perimeter?

h) Determine the volume of a cylindrical can with radius 3 centimetres and height 2 centimetres.

i) Solve $\cos(60°)$.

j) Simplify $2\sin^2(x) + 2\cos^2(x)$.