KUTAISI INTERNATIONAL UNIVERSITY

Prof. Dr. Giorgi Chelidze Prof. Dr. Malkhaz Shashiashvili Prof. Dr. Dr. H.C. Florian Rupp

Antiderivatives & Indefinite Integrals

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, the actual homework assignments are stated. Please, hand-in your results of the homework assignments through MSTeams at the date and time specified in MSTeams.

Additional Exercises (see the lecture slides for solutions):

Exercise 11.1: Find the general indefinite integral.

a)
$$\int \frac{1 + \sqrt{x} + x}{x} dx$$

b)
$$\int (\sin(x) + \sinh(x)) dx$$

c)
$$\int \left(x^2 + 1 + \frac{1}{x^2 + 1}\right) dx$$

d)
$$\int \frac{\sin(2x)}{\sin(x)} dx$$

- **Exercise 11.2:** Find the function f(x) whose tangent has slope $3x^2 + 1$ for each value of x and whose graph passes through the point (2, 6).
- **Exercise 11.3:** A particle moves in a straight line and has acceleration given by a(t) = 6t + 4. Its initial velocity is $v(0) = -6 \ cm/s$ and its initial displacement is $s(0) = 9 \ cm$. Find its position function s(t).
- **Exercise 11.4:** A ball is thrown upward with a speed of 15 m/s from the edge of a cliff 130 m above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?

Homework Assignment:

Problem 11.1: Antiderivatives and indefinite integrals

a) Find the most general antiderivative of the function. (Check your answer by differentiation.)

(i)
$$f(x) = 4x + 7$$

(ii) $f(x) = e^2$
(iii) $f(x) = \frac{2x^2 + 5}{x^2 + 1}$
(iv) $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$
(v) $f(x) = 6x^5 - 8x^4 - 9x^2$
(vi) $f(x) = 2\cos(x) - \frac{3}{\sqrt{1 - x^2}}$



Calculus I for Management

Fall Term Week 11 b) Find the indicated integral. (Check your answer by differentiation.)

(i)
$$\int \left(3x^2 - \sqrt{5x} + 6\right) dx$$

(ii)
$$\int (e^x + 1)^2 dx$$

(iii)
$$\int \left(2e^x + \frac{6}{u} + \ln(2)\right) dx$$

(iv)
$$\int x^3 \left(3x + \frac{1}{x}\right) dx$$

(v)
$$\int \left(3\sqrt{x} - 2x^{-3}\right) dx$$

(vi)
$$\int \frac{x^2 + 3x - 2}{\sqrt{x}} dx$$

- c) Find f
 - (i) $f''(x) = 30x^3 12x^2 + 6x$.
 - (ii) $f''(x) = -2 + 12x 12x^2$, f(0) = 4, and f'(0) = 12.
 - (iii) $f''(x) = \sin(x) + \cos(x)$, f(0) = 3, and f'(0) = 4.
 - (iv) $f'''(x) = \cos(x), f(0) = 1, f'(0) = 2, \text{ and } f''(0) = 3.$
- d) Given that the graph of f passes through the point (2,5) and that the slope of its tangent line at (x, f(x)) is 3 4x, find f(1).
- e) Find a function f such that $f'(x) = x^3$ and the line x + y = 0 is tangent to the graph of f.

Problem 11.2: Applications in business and economics

- a) A manufacturer estimates that the marginal cost of producing q units of a certain commodity is $C'(q) = 3q^2 24q + 48$ GEL per unit. If the cost of producing 10 units is 5000 GEL, what is the cost of producing 30 units?
- b) The marginal profit of a certain commodity is P'(q) = 100 2q when q units are produced. When 10 units are produced, the profit is 700 GEL.
 - (i) Find the profit function P(q).
 - (ii) What production level q results in maximum profit? What is the maximum profit?.
- c) Suppose the consumption function for a particular country is c(x), where x is national disposable income. Then the marginal propensity to consume is c'(x). Suppose x and c are both measured in billions of GEL and

$$c'(x) = 0.9 + 0.3\sqrt{x}$$
.

If consumption is 10 billion GEL when x = 0, find c(x).

- d) A manufacturer estimates marginal revenue to be $200q^{-1/2}$ GEL per unit when the level of production is q units. The corresponding marginal cost has been found to be 0.4q GEL per unit. If the manufacturer's profit is 2000 GEL when the level of production is 25 units, what is the profit when the level of production is 36 units?
- e) A certain oil well that yields 400 barrels of crude oil per month will run dry in 2 years. The price of crude oil is currently 98 USD per barrel and is expected to rise at the constant rate of 40 cents per barrel per month. If the oil is sold as soon as it is extracted from the ground, how much total revenue will be obtained from the well?
- f) Suppose the owner of the oil well in e) decides to step up production so that 600 barrels per month are extracted but everything else remains the same.
 - (i) How many months pass before the well runs dry?
 - (ii) How much total revenue will be obtained from the well?