KUTAISI INTERNATIONAL UNIVERSITY

PROF. DR. GIORGI CHELIDZE PROF. DR. MALKHAZ SHASHIASHVILI PROF. DR. DR. H.C. FLORIAN RUPP

Integration by Parts

This exercise sheet consists of two parts: at first additional exercises are given the solutions of which are provided with the lecture slides and can serve you as further blueprints when solving similar tasks. Then, some suggestions for self-study assignments are stated.

Additional Exercises (see the lecture slides for solutions):

Exercise 15.1: Evaluate the following integral using integration by parts by using the choice $u = \ln(x)$ and $dv = \sqrt{x} dx$:

$$\int \sqrt{x} \cdot \ln(x) \, \mathrm{d}x$$

Exercise 15.2: Use integration by parts to prove the reduction formula

$$\int (\ln(x))^n \, \mathrm{d}x = x \cdot (\ln(x))^n - n \int (\ln(x))^{n-1} \, \mathrm{d}x$$

Exercise 15.3: If f(0) = g(0) = 0 and f'' and g'' are continuous, show that

$$\int_0^a f(x)g''(x) \, \mathrm{d}x = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x) \, \mathrm{d}x$$

Exercise 15.4: Evaluate the following integrals using integration by parts:

a)
$$\int_{1}^{5} \frac{x}{e^{x}} dx$$

b) $\int (1+x^{2}) \cdot e^{3x} dx$
c) $\int_{0}^{1} \frac{x^{3}}{\sqrt{4+x^{2}}} dx$
d) $\int e^{x} \sin(\pi x) dx$

Hint: for parts b) and d) applying integration by parts twice may be necessary.



Fall Term Self Study Week 15

Problem 15.1: Integration by parts

a) Evaluate the indefinite integrals:

(i)
$$\int x \cos(5x) dx$$
.
(ii) $\int x^4 \ln(x) dx$.
(iii) $\int x^4 \ln(x) dx$.
(iv) $\int (\ln(x))^2 dx$.

b) Evaluate the definite integrals:

(i)
$$\int_{1}^{5} \frac{\ln(x)}{x^{2}} dx.$$
 (iii) $\int_{0}^{\pi/3} \sin(x) \ln(\cos(x)) dx$
(ii) $\int_{0}^{1} (x^{2} + 1)e^{-x} dx.$ (iv) $\int_{0}^{t} e^{x} \sin(t - x) dx.$

c) First make a substitution and then use integration by parts to evaluate the integral.

(i)
$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} x^3 \cos(x^2) dx$$
, and (ii) $\int \cos(\ln(x)) dx$

- d) After t hours on the job, a factory worker can produce $100te^{-0.5t}$ units per hour. How many units does the worker produce during the first 3 hours?
- e) Joel Evans, the manager of a national chain of pizza parlors, is selling a 6-year franchise to operate its newest outlet in Orlando, Florida. Experience in similar localities suggests that t years from now, the franchise will be generating profit continuously at the rate of R(t) = 300 + 5t thousand dollars per year. If the prevailing rate of interest remains fixed during the next 6 years at an annual rate of 6% compounded continuously, what would be a fair price for Joel to charge for the franchise? [Hint: Use present value to measure what the franchise is worth.]



We Wish You a Merry Christmas and a Happy New Year